Solid theory

$$iG(x_a, t_a; x_b, t_b) = \left(\frac{m}{2\pi i \Delta t}\right)^{N/2} \int dx_1 \cdots dx_N \, e^{i\int_{t_0}^{t} dt}$$

$$\left(\frac{m}{2}\dot{x}^2 - V(x)\right)$$

$$x_c(t) = x_a + \frac{x_b - x_a}{t_b - t_a}(t - t_a)$$

$$\int_{t_a}^{t_b} \mathcal{L}\, dt = \frac{m}{2}\dot{x}^2 (t_b - t_a) = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}$$

Fluctuations: $\delta x_j = x_j - x_c(t_j)$

Free space:

$$iG(x_a, t_a; x_b, t_b) = \left(\frac{m}{2\pi i \Delta t}\right)^{N/2} \int dx_1 \cdots\cdots dx_{N-1}$$

$$\prod_{i=1}^{N} \exp\left(i m \frac{(x_i - x_{i-1})^2}{\Delta t}\right)$$

$$\downarrow$$

$$\frac{(x_c(i) - x_c(i-1))^2}{\Delta t} + \frac{(\delta x_i - \delta x_{i-1})^2}{\Delta t}$$

③

$$\Rightarrow i\, G(x_a, t_a; x_b, t_b) = \left(\frac{m}{2\pi i \Delta t}\right)^{N/2} e^{iS_c} \int d\delta x_1\, d\cdots \delta x_{N-1}$$

$$\prod_{i=2}^{N-1} e^{i\frac{m}{2}(\delta x_i - \delta x_{i-1})^2/\Delta t}$$

$$\Downarrow$$

$$\exp\Big(i \sum_{j,k} \delta x_j\, M_{jk}\, \delta x_k\Big)$$

$$M_{jk} = \begin{pmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ 0 & -1 & 2 & -1 \\ & & & \end{pmatrix}$$

$$M_N = \begin{pmatrix} 2\cosh u & -1 & & \\ -1 & 2\cosh u & -1 & \\ & & \ddots & \\ & & & 2\cosh u \end{pmatrix} = \frac{\sinh(N+1)u}{\sinh u}$$

$$= N+1$$

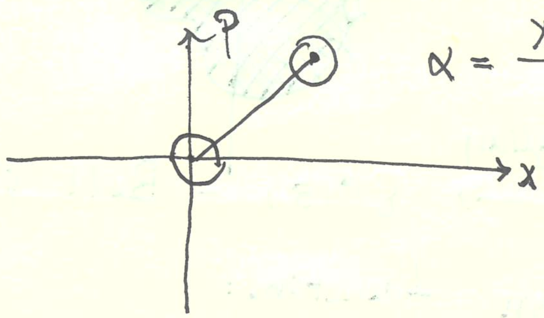
$$= \left(\frac{m}{2\pi i \Delta t}\right)^{N/2} e^{iS_c} \left(\frac{m}{2\pi i \Delta t}\right)^{-\frac{N-1}{2}} N^{-1/2}$$

$$= \left(\frac{m}{2\pi i (t_b - t_a)}\right)^{1/2} e^{iS_c}$$

· Path integral on the phase space

Re Squeeze operator: $e^{\alpha a^+ a^+}$

Physical meaning



$\alpha = \frac{x + ip}{\sqrt{2}}$

$$\int \frac{\mathrm{Re}(\alpha)\,\mathrm{Im}\,\alpha}{\pi} |\alpha\rangle\langle\alpha| = 1$$

$$iG(\alpha_b t_b, \alpha_a t_a) = \int \frac{d^2\alpha_1 \cdots d^2\alpha_{N-1}}{\pi^{N-1}} \, iG(x_b, t_b, x_{N-1} t_{N-1})$$

$$\cdots \, iG(x_1 t_1; x_a t_a)$$

$$G(\alpha_i, t_i; \alpha_{i-1} t_{i-1}) \equiv \langle\alpha_i| e^{-i\omega\Delta t\, a^+ a} |\alpha_{i-1}\rangle$$

$$\equiv e^{-i\Delta t\, \bar{\alpha}_i \alpha_{i-1} - \frac{1}{2}\alpha_i^*(\alpha_i - \alpha_{i-1}) + \frac{1}{2}\alpha_{i-1}(\alpha_i^* - \alpha_{i-1}^*)}$$

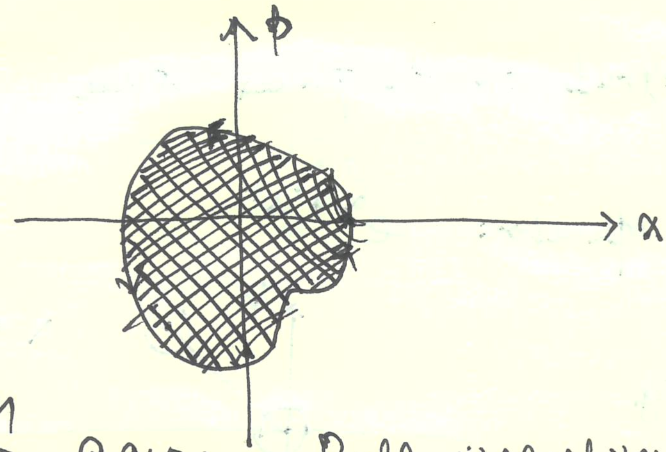
$$\equiv e^{-i\omega\Delta t\, \alpha_i^* \alpha_i + \frac{i}{2}(\alpha_i^* \dot{\alpha}_i - \alpha_i \dot{\alpha}_i^*)}$$ Berry phase

$$\equiv \int \frac{\prod_{i=1}^{N-1} d^2\alpha_i}{\pi^{N-1}} \exp\left(i \int_{t_a}^{t_b} \frac{i}{2}(\alpha^* \dot{\alpha} - \alpha \dot{\alpha}^*) - \underbrace{\omega \alpha^* \alpha}_{H}\right)$$

$$\Downarrow$$

$\langle\alpha| \partial_t |\alpha\rangle$ $\quad p\dot{x}$

④



Nonabelian → $SO(5)$ – Bell inequality

- Partion function:

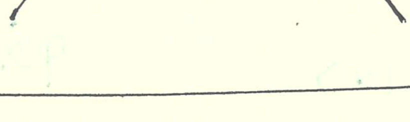
$$Z = \mathrm{tr}(e^{-\beta H}) \equiv \int D(\tau)\, e^{-\int_0^\beta \left(\frac{i}{2}\alpha^* \dot{\alpha} - \alpha\alpha^* - \omega\alpha^*\alpha\right)}$$

$$\alpha = \frac{x + ip}{\sqrt{2}} \qquad \alpha^+ = \frac{x - ip}{\sqrt{2}}$$

贝尔不等式 → 经典的不等式概率是正的

量子力学可以是Negative的，也可是Complex的

用Path integral去搞Bell不等式



Operator formalism, response function:

$$|m_1, m_2, m_3, m_4\rangle \equiv (\Phi_1^{m_1})(\Phi_2)^{m_2}\cdots(\Phi_4)^{m_4}\,|0\rangle$$

- Perturbation theory:

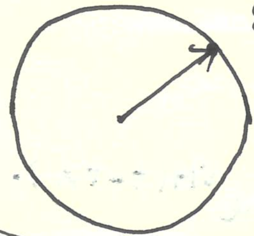
$$H + f(t)\,Q \qquad t = -\infty,\ f(t) \to 0$$

$$|\Phi_n(t)\rangle \equiv T\, e^{-i\int_{-\infty}^{t} dt'\,(H + f(t)\,O_i)}\,|\Phi_n\rangle$$

Kubo formula

↓

如何推广到 Nonhermitian



S_3

$$g\,\partial_\mu g^{-1}$$

$$g = \begin{pmatrix} x_0 + x_3 & x_1 - i x_2 \\ x_1 + i x_2 & x_0 - x_3 \end{pmatrix}$$

$$\frac{1}{24\pi^2}\,\mathrm{tr}\left(\epsilon_{\mu\nu\lambda}\, g\,\partial_\mu g^{-1}\,\partial_\nu g\,\partial_\lambda g^{-1}\right)$$

$$T\left(e^{-i\int_{-\infty}^{t} dt'\,(H + f(t)\,O)}\right) \equiv T\Big(e^{-i\int_{-\infty}^{t} dt'\,H}$$

$$\left(1 + (-i)\int_{-\infty}^{t} dt''\, f(t'')\, O + \cdots\right)\Big)$$

$$H = \left(\frac{\partial}{\partial u}u + \sqrt{\frac{\partial}{\partial v}}v\right)\left(u\frac{\partial}{\partial u} + 0\,\frac{\partial}{\partial v}\right) - \left(\frac{eg}{c}\right)^2$$

$$\delta\Psi_n \equiv -i\int_{t_\infty}^{t} dt'\, e^{-iH(t-t')}\left(f(t')\,O_1\right) e^{-iH(t-t_\infty)}\,|\Psi_n\rangle$$

$$\equiv -i\int_{t_\infty}^{t} dt'\, f(t')\, e^{-iH(t-t_\infty)}\, O_1(t')\,|\Psi_n\rangle$$

$$O_1(t') = \underbrace{e^{iH(t'-t_\infty)}\, O_1\, e^{-iH(t-t_\infty)}}_{\text{interaction picture}}$$

- $$\delta O_2(t) \equiv \langle\Psi_n(t)|\,O_2\,|\Psi_n(t)\rangle - \langle\Psi_n|\, e^{iH(t-t_\infty)}$$

$$O_2\, e^{-iH(t-t_\infty)}\,|\Psi_n\rangle$$

$$\equiv -i\int_{t-\infty}^{t} dt'\, \langle\Psi_n|\, \underbrace{e^{iH(t-t_\infty)}\, O_2\, e^{-iH(t-t_\infty)}}_{2}\, \underbrace{f(t')}_{1}$$

$$\underline{O_1(t')}\,|\Psi_n\rangle \qquad - \quad (2-1)$$

$$\equiv -i\int_{t_\infty}^{t} \langle\Psi_n|\,[O_2(t),\ O_1(t')]\,|\Psi_n\rangle\, f(t')$$

$$\delta O_2(t) = \int_{-\infty}^{+\infty} dt'\, D(t-t')\, f(t')$$

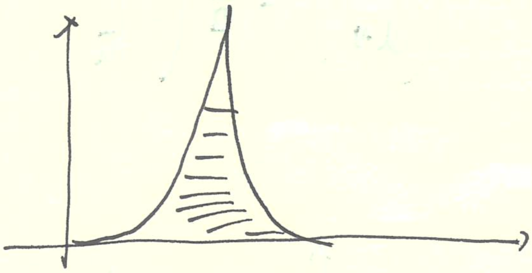
$$D(t-t') = -i\,\Theta(t-t')\,\langle\,|\,[O_2(t)-,\ O_1(t')]\,\rangle$$

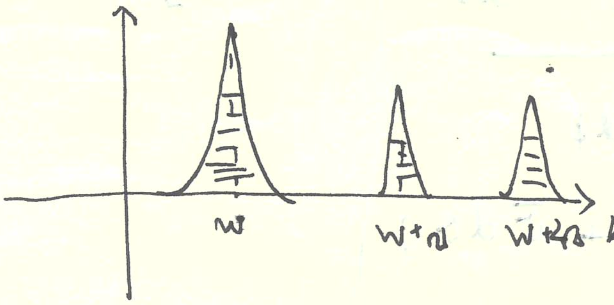
还有更深刻的意义

$H(t) = H(\pm) + f(t)\,O$

Linear theory 都是建立在原来系统不含时的基础上:

若 $H \to H(t)$ 或 $V(x,t)$





就是说一束激光打进去之后 $\omega+\Omega$, $\omega+2\Omega$, $\omega+4\Omega$